Session 1

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September 5, 2026

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What are we doing here?

- I want to do whatever would be the most helpful for you guys.
- Focusing on W/Th of previous week and M/T content of current week.

Possible thoughts:

- Reinforce lecture content
- You do some problems
- I go through some problems
- Anything else?

I will always have some time at the beginning to answer any questions.

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Theorem
If f is non-negative monotone decreasing, then $\sum_{i=1}^{n} f(i) = \Theta(\int_1^n f(x)dx)$

Proof.
Main idea is as follows:

$$\int_1^n f(x)dx \leq \sum_{i=1}^{n} f(i) \leq f(1) + \int_1^n f(x)dx$$

$\square$

On monotone increasing

For monotone increasing we have:

$$\int_1^n f(x)dx \leq \sum_{i=1}^{n} f(i) \leq \int_1^n f(x)dx + f(M)$$

So as long as $f(M) \in O(\int_1^n f(x)dx)$, we have $\sum_{i=1}^n f(i) \in \Theta(\int_1^n f(x)dx)$.

This covers all non-super exponential functions

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- $\sum_{i=1}^{n} i^\rho, \rho < -1 \quad \Theta(1)$

Master's Theorem is Cringe

- I **personally** don't see much value in it

- Pretty limited in its capabilities

- You should be able to calculate these recursions on the spot

Practice with Solving Recursive Equations

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- $T(n) = 2T(\sqrt{n}) + O(1) \qquad O(\log(n))$
- $T(n) = 4T(n/3) + O(n), \quad O(n^{1.262})$

Solving $T(n) = 2T(n/2) + O(n^3)$

First level of tree has $O(n^3)$ work, second level has 2 calls with $(n/2)^3$ work.[1]

$$\begin{aligned} T(n) &\ll \sum_{i=1}^{\log_2(n)} (1/4)^i n^3 \\ &\ll n^3 \sum_{i=1}^{\log_2(n)} (1/4)^i \\ &\ll n^3 \end{aligned}$$

[1] $f(x) \ll g(x)$ means $f(x) \in O(g(x))$

Solving $T(n) = 2T(\sqrt{n}) + O(1)$

First level does c work, second level does $2c$ work, third does $4c$ work and so forth.

Question
How many levels are there in the recursion tree?

The input size at each level decreases by $\sqrt{\bullet}$, let k be the tree depth then:

$$\begin{aligned} n^{1/2^k} &\leq C \\ 1/2^k \cdot \log(n) &\leq \log(C) \\ -k + \log\log(n) &\leq \log\log(C) \\ k &\geq \log\log(n) - \log\log(C) \end{aligned}$$

Solving $T(n) = 2T(\sqrt{n}) + O(1)$ cont.

$$
\begin{aligned}
T(n) &\ll \sum_{i=0}^{\log\log(n)} 2^i \\
&\ll \int_0^{\log\log(n)} 2^x dx \\
&\ll 2^{\log\log(n)} \\
&\ll \log(n)
\end{aligned}
$$

Thus $T(n) \in O(\log(n))$

Solving $T(n) = 4T(n/3) + O(n)$

$$T(n) \ll \sum_{i=0}^{\log_3(n)} (4/3)^i n$$

Solving $T(n) = 4T(n/3) + O(n)$

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\begin{aligned}
T(n) &\ll \sum_{i=0}^{\log_3(n)} (4/3)^i n \\
&\ll n \sum_{i=0}^{\log_3(n)} (4/3)^i
\end{aligned}
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&\ll n \sum_{i=0}^{\log_3(n)} (4/3)^i \\
&\ll n \int_0^{\log_3(n)} (4/3)^x \, dx \\
&\ll n(4/3)^{\log_3(n)}
\end{aligned}
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&\ll n(4/3)^{\log_3(n)} \\
&\ll n(4/3)^{\log_{4/3}(n)/\log_{4/3}(3)} \quad \log_b(a) = \frac{\log_c(a)}{\log_c(b)}
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&\ll n(4/3)^{\log_{4/3}(n)/\log_{4/3}(3)} \qquad \log_b(a) = \frac{\log_c(a)}{\log_c(b)} \\
&\ll n \cdot n^{1/\log_{4/3}(3)} \\
&\ll n^{1.26}
\end{aligned}
$$

Recursion Practice

- Being able to think recursively is crucial in this class (e.g. D&C and DP)

Problem
Given an array of integers, $A = [1..n]$, with $A[1]$ odd and $A[n]$ even, find an index i such that $A[i]$ is odd and $A[i+1]$ is even.

Problem
(Subset Sum) Given n integers $x_1, \ldots, x_n \in \mathbb{Z}$ and an integer T, determine if there is a subset of indices $S \subseteq [n]$ such that $\sum_{i \in S} x_i = T$. We don't care about runtime, just want you to solve recursively.