

Session 7

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Short Rant on Point Packing

Suppose you have some (bounded, sensible) set $K \subseteq \mathbb{R}^2$ and you want to upper bound the number of points you can place in K such that the distance of any two points is at least δ .¹

Intuitively, you can think of each point as being surrounded by a disk of radius $\delta/2$, and these disks should not overlap. Then the total area covered by these disks is at most the area of $K + B_{\delta/2} = \{x + y : x \in K, y \in B_{\delta/2}\}$.

Then the number of points you can place is at most the area of $K + B_{\delta/2}$ divided by the area of a single disk of radius $\delta/2$.

¹Note this can easily be extended to $\mathbb{R}^d$ for any d .

Example

For example on your homework, you wanted to know the max number of points you could place in a $3\delta \times 4\delta$ rectangle, K , with distance at least δ between any two points. The area of $K + B_{\delta/2}$ is $< (3\delta + \delta) \cdot (4\delta + \delta) = 4\delta \cdot 5\delta = 20\delta^2$. The area of a single disk of radius $\delta/2$ is $\pi(\delta/2)^2 = \pi\delta^2/4$. So the maximum number of points you can place is at most $\lfloor \frac{20\delta^2}{\pi\delta^2/4} \rfloor = \lfloor \frac{20 \cdot 4}{\pi} \rfloor = \lfloor \frac{80}{\pi} \rfloor = 25$. Which is sufficient for the algorithm runtime since you only need a constant.

Another example is if you take K to be a unit disk. The area of $K + B_{\delta/2}$ is $\pi(1 + \delta/2)^2$. So the maximum number of points you can place is at most $\lfloor \frac{\pi(1+\delta/2)^2}{\pi\delta^2/4} \rfloor = \lfloor \frac{4(1+\delta/2)^2}{\delta^2} \rfloor$.

Advice For the Exam

If you want to use any algorithm from class, use it as a black box
Do not reimplement an algorithm from class unless asked to do so explicitly.²

Although a problem might not say to give a proof of correctness, I still encourage you to try and prove it, as that will tell you if your solution is correct.

Write your pseudocode as simple as possible. For example, if you want to find the min of an array just say “find the min”, rather than writing out a full loop to do so.

²I will cry if I see someone reimplement k -selection.

Advice For the Exam cont.

If you can't figure out a problem just skip it and come back later. I find that I think about the problems in the background of solving other problems.

If you are writing more than a page on a problem, you're probably writing too much.

Any questions for me?